Fluids Quiz 1 Solution - Problem 2 S'08

Given: $\gamma(\theta) = V_\infty \left[1.0 \sin\theta + 0.4 \sin 2\theta \right]$

General case: $\gamma(\theta) = 2V_\infty \left[A_0 \frac{1+\cos\theta}{\sin\theta} + A_1 \sin\theta + A_2 \sin 2\theta + \dots \right]$

$\therefore \quad A_0 = 0 \;,\; A_1 = 0.5 \;,\; A_2 = 0.2 ,\; A_3 = A_4 = \dots = 0$

a) $\boxed{c_\ell = \pi \left[2A_0 + A_1 \right] = 0.5\,\pi = 1.78}$

$\boxed{c_{m_{c/4}} = \frac{\pi}{4}\left[A_2 - A_1 \right] = -0.3\,\frac{\pi}{4} = -0.2356}$

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Alternative: (the long way) $\Delta p = \rho V_\infty \gamma = \rho V_\infty^2 \left[1.0 \sin\theta + 0.4 \sin 2\theta \right]$

$$L' = \int_0^c \Delta p \, dx = \int_0^\pi \Delta p \, \frac{c}{2} \sin\theta \, d\theta = \int_0^\pi \rho V_\infty^2 \frac{c}{2} \left[1.0 \sin\theta + 0.4 \sin 2\theta \right] \sin\theta \, d\theta$$

$$L' = \tfrac{1}{2}\rho V_\infty^2 c \cdot 1.0 \cdot \frac{\pi}{2} \rightarrow c_\ell = \frac{L'}{\frac{1}{2}\rho V_\infty^2 c} = \frac{\pi}{2} = 1.78$$

$$M'_{c/4} = \int_0^c \Delta p \cdot \left(\frac{c}{4} - x\right) dx = \int_0^\pi \Delta p \left(\frac{c}{4} - \frac{c}{2}(1-\cos\theta) \right) \frac{c}{2} \sin\theta \, d\theta$$

$$= \int_0^\pi \rho V_\infty^2 \frac{c^2}{2} \left[1.0 \sin\theta + 0.4 \sin 2\theta \right] \left(-\frac{1}{4} + \frac{1}{2}\cos\theta \right) \sin\theta \, d\theta$$

$$= \int_0^\pi \rho V_\infty^2 \frac{c^2}{2} \left[1.0 \sin\theta + 0.4 \sin 2\theta \right] \left[-\frac{1}{4}\sin\theta + \frac{1}{4}\sin 2\theta \right] d\theta$$

$$M'_{c/4} = \tfrac{1}{2}\rho V_\infty^2 c^2 \left[-0.25\frac{\pi}{2} + 0.1\frac{\pi}{2} \right] = \tfrac{1}{2}\rho V_\infty^2 c^2 \left(-0.15\frac{\pi}{2} \right) \rightarrow c_{m_{c/4}} = \frac{M'}{\frac{1}{2}\rho V_\infty^2 c^2} = -0.2356$$

b) A_n's are also related to airfoil slope:

$$\alpha - \frac{dz}{dx} = A_0 - \sum A_n \cos n\theta$$

$$\frac{dz}{dx} = \alpha - A_0 + \sum A_n \cos n\theta = \overbrace{\alpha + 0.5 \cos\theta + 0.2 \cos 2\theta}^{\text{for this case}}$$

At $\frac{x}{c} = \frac{1}{2}$ (midchord), $\theta = \frac{\pi}{2}$, $\cos\theta = 0$, $\cos 2\theta = -1$

$$\therefore \; \boxed{\frac{dz}{dx} = \alpha - 0.2 = 0.1 - 0.2 = -0.1}$$

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